Material Point Method and Applications in Geotechnical Engineering

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Introduction

A vast number of computational methods is being developed to simulate large deformation problems involving soil-water-structure interaction. Here, the material point method (MPM) is used that was developed to simulate large deformations in history-dependent materials. It combines the advantages of mesh-based and point-based approaches: mesh distortion is eliminated and history is stored in material points. Generally speaking, MPM is an advancement of the finite element method (FEM) where the continuum body is represented by a set of Lagrangian points, so-called material points (MPs). The MPs are moving through an Eulerian background mesh. The MPs carry all physical properties of the continuum such as stresses, strains, density, momentum, material parameters and other state parameters, whereas the background mesh is used to solve the balance equations without storing any permanent information.

Additional benefit of MPM is the continuum description of the material for which well-known constitutive relations to describe the material's stress-strain behaviour can be applied. Furthermore, the application of engineering boundary conditions is easily facilitated, i.e. stresses and displacements or their rates can directly be applied on any boundary or material point. Shortcomings of MPM are its mesh-dependency which is inherent to any finite element formulation,

computational costs and stability caused by material points crossing element boundaries. An overview of the historical development and applications of MPM can be found in several contributions of the proceedings of the first international conference on the material point method (Procedia Engineering, 2017). They provide a state-of-the-art overview of currently active groups developing MPM and recent advancements in applications for geotechnical and hydraulic engineering.

The material point method is implemented into the Anura3D MPM Software of the Anura3D MPM Research Community (www.anura3d.com). This is a collaboration of the Universitat Politècnica de Catalunya (UPC Barcelona), University of California Berkeley, University of Cambridge UK, Delft University of Technology, Deltares, Università degli Studi di Padova, Technische Universität Hamburg-Harburg (TUHH) and associated partner Università degli Studi di Salerno.

Concept of Material Point Method (MPM)

In MPM the computational domain is spatially discretised in two frames. First, the continuum body is divided into a set of material points (MPs). Each MP represents an initially defined volume of the domain with associated mass. One of the basic and most important features of MPM is that the mass of each MP strictly remains constant which implies that mass conservation is automatically satisfied, whilst the

volume of the MP can change enabling material compression or extension. The second frame is the computational background mesh which is equivalent to a conventional finite element mesh. It is constructed to cover the full domain of the problem, i.e. also areas where material is expected to move into during computation. The discretised momentum balance equations are solved at the nodes of this computational mesh, whereas mass conservation and constitutive equations are solved at the MPs.

The MPM algorithm for a single calculation step is illustrated in Figure 1. At the beginning of each step, the components of the momentum balance equations are defined by mapping information from the MPs to the nodes of the computational mesh by means of interpolation or shape functions (Fig. 1a). The equations of motion are solved for the primary unknown variables, i.e. the nodal accelerations (Fig. 1b). These nodal values are used to update acceleration, velocity and position of MPs, as well as to compute strains and stresses at the MPs (Fig. 1c). The assignment of MPs to elements is updated after mesh adjustment (Fig. 1d). For technical details the reader is referred to e.g. Rohe & Martinelli (2017).

Soil, in general, is a mixture of three constituents (solid, liquid and gas) which interact with each other determining the mechanical and hydraulic response of the material. However, taking rigorously into account these interactions
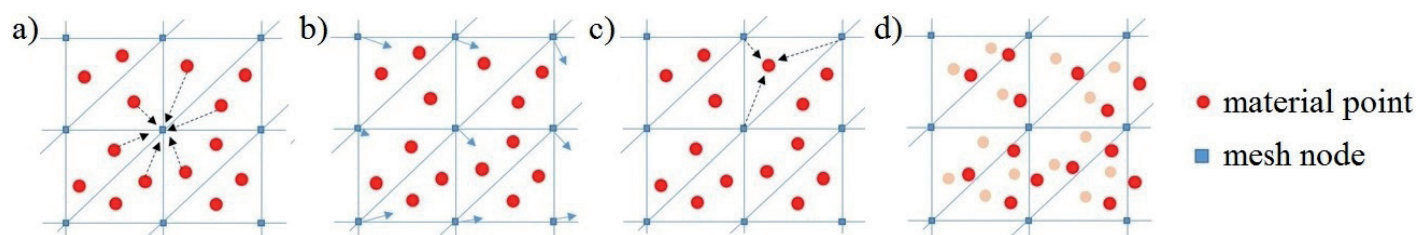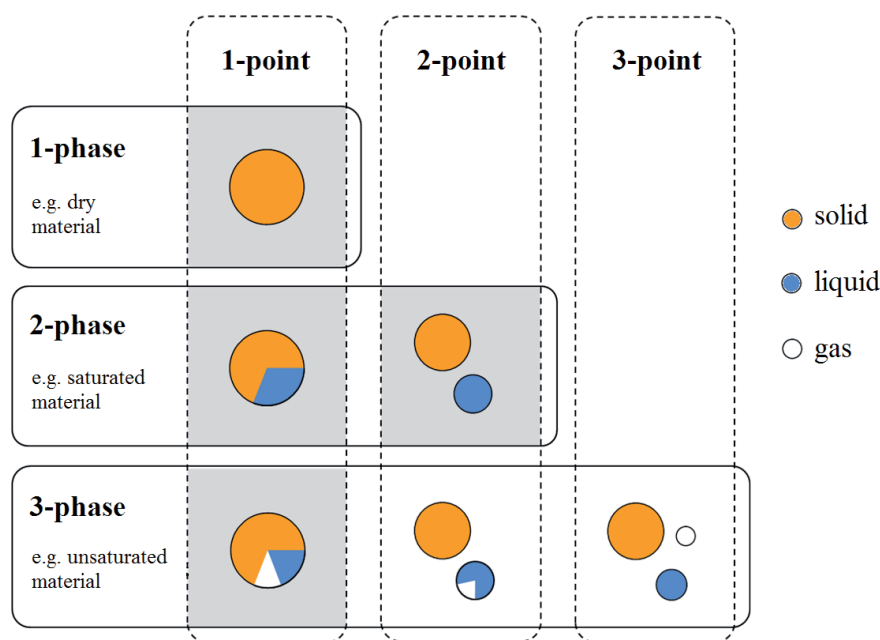
Adapted from MPM Research Community, 2017

Figure 1 - *Computational scheme of MPM.*

a)   b)   c)   d)

● material point
■ mesh node

Abstract

The concept of the material point method (MPM) is briefly introduced for the numerical modelling of large deformations and soil-water-structure interaction for applications in geotechnical and hydraulic engineering. The formulation of MPM for a fully-coupled two-phase porous material in contact and interaction with free surface water is used. The concept of multi-ple sets of material points is introduced which enables modelling of free surface water, groundwater flow, liquefaction and erosion problems. The numerical framework is implemented in the Anura3D MPM Software. Two application examples are presented, i.e. slope failure due to infiltration and due to seepage flow.

Figure 2 - Concept of multiple constituents in MPM approaches depending on number of phases and number of material point sets as used in Anura3D.

Adapted from Yerro et al., 2015

Figure 3 - Example of the 2-point MPM schematisation of a partly saturated slope in contact and interaction with free surface water.

Adapted from Martinelli, 2016

fields of applicability of the 1-phase 1-point formulation. In this case only one set of material points is needed to carry all required information of the material. Typical applications are undrained slope failure, collapse of dry granular materials, silo discharge problems, water reservoirs, shallow foundations in undrained conditions, undrained cone penetration tests (CPTs) and submerged slope failure in highly permeable sandy soils. The full formulation of the 1-phase 1-point approach can be found in e.g. MPM Research Community (2017).

In the 2-phase 1-point approach each material point describes a representative volume element of fully saturated soil, carrying the information of both phases, liquid and solid together. While the material points are attached to the solid skeleton giving a Lagrangian description of its movement, an Eulerian approach with respect to the solid represents the motion of the liquid phase. Typical applications are consolidation, pile installation (jacking, driving, vibrating) with generation and dissipation of excess pore pressures, CPTs in partially drained conditions and failure of saturated slopes due to infiltration or loading. A full formulation of the 2-phase 1-point approach can be found in MPM Research Community (2017).

In the 3-phase 1-point approach, the soil is understood as a material composed of three distinct constituents, i.e. solid, liquid and gas. The solid phase constitutes the soil skeleton of the media while liquid and gas phases fill the voids. All phases are combined in a single material point and balance and momentum equations are formulated and numerically solved as above. No independent motion of the water and air is expected in the applications envisaged. Typical applications are rainfall and drought effects in slope failure and collapse analyses for unsaturated soils, collapse behaviour of low-density soils and unrestrained swelling of expansive clays. The full formulation of the 3-phase 1-point approach can be found in Yerro et al. (2015).

In the 2-phase 2-point approach the solid-liquid mixture is modelled using two distinct sets of
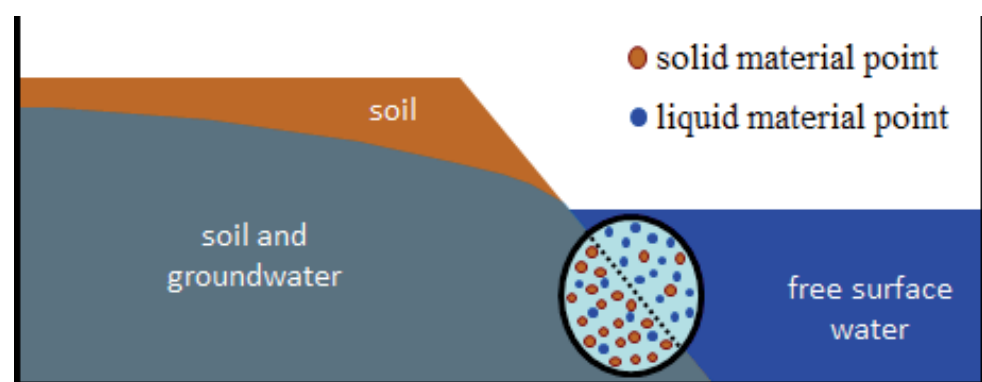
may be in many cases unnecessarily complicated, computationally expensive, and even unfeasible for engineering applications. In Figure 2 the concept of multiple constituents as used in the MPM formulation of Anura3D is shown. The rows represent the number of phases of the continuum, the columns represent the number of material point sets to describe each phase. The grey shaded combinations are currently available in the MPM implementation of Anura3D and are described below.

Modelling of dry soil, pure liquid and saturated soil in drained and undrained conditions are the

Figure 4 - *Anura3D results for modelling slope failure due to infiltration. Evolution of a) equivalent plastic shear strain; b) excess pore water pressures [kPa]; for time t* in whole slope. Evolution of c) total displacements [m]; d) velocity [m/s]; in four points of the slope.*

Adapted from Soga et al., 2016
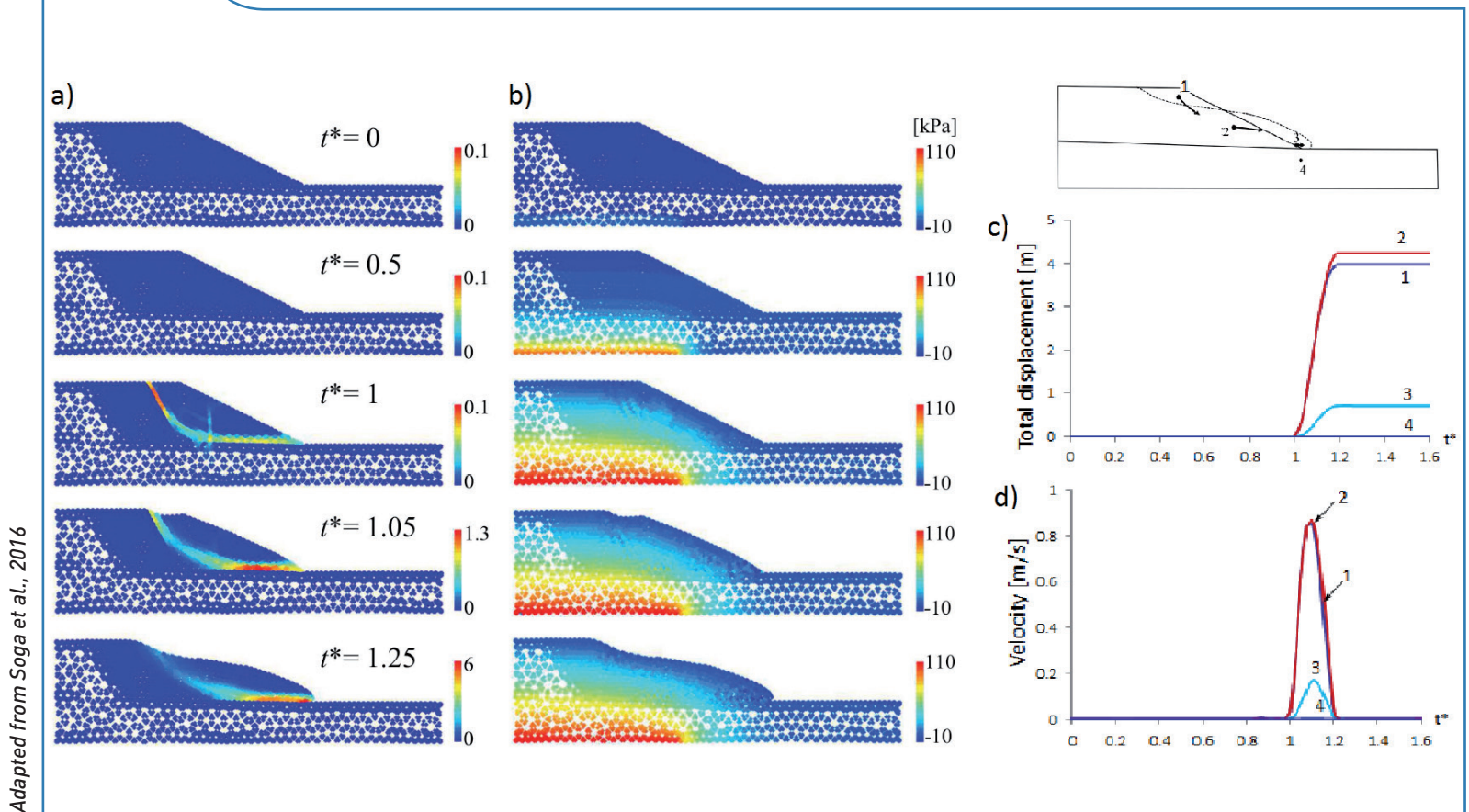
material points, one for each constituent respectively. Figure 3 shows an example of a slope which is characterised by two materials: soil (dry and saturated parts) and free surface water. The soil is a porous material composed of two constituents, the solid skeleton and groundwater respectively, whereas the free surface water is a pure liquid. In general, the water can flow out of the soil body into the water reservoir or vice-versa, and interacts with the solid skeleton through drag forces. The soil can behave as a solid porous material with liquid in its pores (solid-like response) or as a liquefied material in which soil grains float in the liquid (liquid-like response). The material can also change between these states. Application examples are transient seepage flow through a porous material or the free-fall of a poro-elastic body under water (e.g. dropping of geocontainers). Advanced examples involving state transition are the free-fall of a coarse-grained soil under water, collapse of a submerged sand column (liquefaction and breaching), collapse of a slope due to seepage flow with subsequent sediment transport, internal instability (suffusion), piping, erosion problems, slurry and debris flows including separation. The full formulation of the 2-phase 2-point approach can be found in Martinelli (2016).

Application 1: Slope failure due to infiltration

The 2-phase 1-point formulation is used to analyse the failure of a slope due to water infiltration below the dike. The Selborne slope failure experiment (Cooper, 1998) is simulated in which progressive failure in overconsolidated clays was investigated in nearly fully saturated conditions. The instability process is illustrated in Figure 4. The evolution of equivalent plastic strain and excess pore water pressures is shown at five normalised times $t*=t/T_{failure}$. Note that the plastic shear strain scale varies in order to capture the results properly. The calculated total displacements and velocity of four points along the slope are shown.

The progressive nature of the slope failure can be observed. The slope remains stable until $t*=1$. When the failure mechanism develops those material points located above the shear band (1, 2, 3) accelerate quickly and a peak velocity is attained at $t*=1.1$. At $t*=1.2$ the slope tends to stabilise. Point 4 remains motionless and point 3 at the toe moves only a small distance. The development of failure starting at the toe and crest of the slope, geometry of the failure surface, measured displacements of the slope and the time history of pore pres-

sures are successfully simulated using the 2-phase 1-point MPM formulation in Anura3D. It is shown that MPM can adequately simulate the problem, and results compare well with field measurements of the Selborne experiment. Further details can be found in Soga et al. (2016).

Application 2: Slope failure due to seepage flow

The 2-phase 2-point formulation is used to analyse the failure of a dike slope due to seepage flow. During the calculation the water head in the upstream reservoir (left) drops and water flows through the slope towards the downstream reservoir. Figure 5 shows the soil porosity at the location of the solid material points together with the position of the liquid material points at four time steps. The porosity varies between 40% (initial value) and 100% (pure water) but is only shown in the range below the maximum soil porosity of 52%. During the simulation the pore pressure at the toe increases and the drag forces by the flowing water move the solid material points rightwards. Plastic shear strains are induced in the soil and the porosity increases accordingly, which is due to the positive dilatancy angle assumed in the elastoplastic Mohr-Coulomb soil constitutive model.

At the beginning (t=6s), the porosity increases only at the toe whereas, as the simulation proceeds (t=8s), the change in porosity propagates also along the shear plane. At the toe the porosity increases until it exceeds the limit porosity and fluidises. As the time progresses (t=14s), the fluidised zone increases, moving upwards towards the crest, and the fluidised material located at the toe is moving into the downstream reservoir. A large increase in porosity is also observed along the failure surface (t=23s), where some solid material points exhibit a fluid-like behaviour. This evidence is related to the basic constitutive model which has a constant dilatancy angle independent of the plastic strains. Therefore, along the failure surface where most of the shear strains are concentrated, the response of the constitutive model produces a progressive increase in volume which leads to fluidisation of the soil. In future analyses a more advanced critical state constitutive model is necessary for modelling more realistic soil behaviour.

Conclusion
The material point method (MPM) is introduced and used for simulating large deformation problems, and there seems an opportunity to utilise this technique to assess the risk of damage after progressing failure. MPM was selected as preferred choice, primarily because: (a) the implementation is intuitive for users of FEM; (b) it can incorporate advanced history-dependent soil constitutive models; (c) its application of boundary conditions is more straightforward than other mesh-free methods owing to the presence of the background grid; and (d) fully coupled behaviour of soil, pore water and free surface water can be included in the formulation. The example applications demonstrate the potential opportunities of MPM for large-deformation soil-water-structure interaction analysis to predict both pre- and post-failure behaviour. The authors would like to thank Jurjen van Deen (Deltares) for his assistance in editing this paper.

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Figure 5 - *Anura3D results for modelling slope failure due to seepage flow. Position of liquid material points (blue coloured) and soil porosity of solid material points (colour legend) is shown for several time steps.*
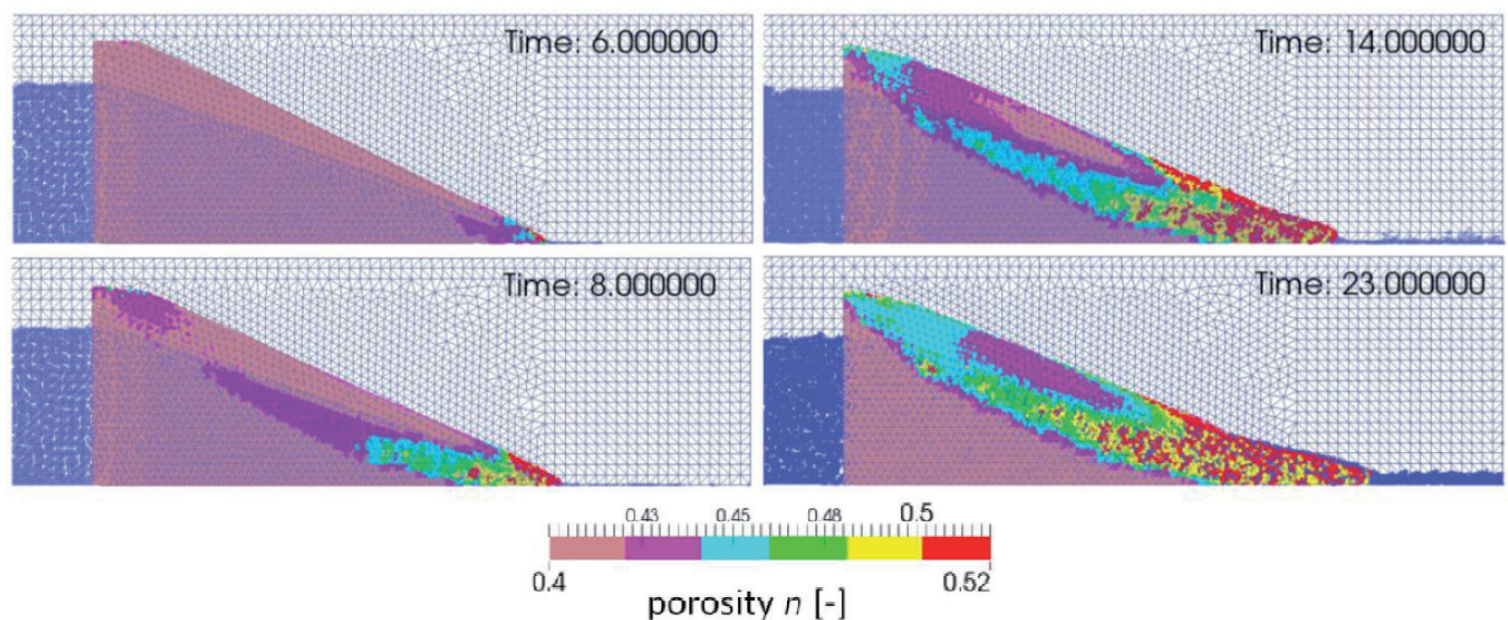
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