INTRODUCTION
The design engineer, André Coyne of the well-established engineering partnership of Coyne et Bellier based in Paris, was nearing the end of his professional career and had over one hundred successful dams constructed. Despite the calamity and the first arch dam ever to fail, his immediate reaction was to try and establish the cause of the failure (Bellier, 1977).

By 1969, ten years later, publications on the method of analysis Coyne and Bellier had established started to appear in the international journals, starting with two papers in the Journal of Soil Mechanics and Foundation Engineering of the ASCE (American Society of Civil Engineers). These were by Pierre Londe, Gaston Vigier and Raymond Vormeringer (1969 and 1970). Subsequently, Pierre Londe published a number of papers in the Quarterly Journal of Engineering Geology (1973) and in Engineering Geology (1987). Both these publications were milestones: The QJEG providing an introduction to rock mechanics as is still practiced today, notably the paper in that issue on slope wedge analysis by Hoek, Bray and Boyd. The Elsevier Engineering Geology publication was based on a meeting held to discuss dam failures.

The milestone QJEG publication re-appeared as part of Hoek and Bray’s Rock Slope Engineering almost the same year and re-appeared recently in a new 4th edition version by Wyllie & Mah (2004). The work by Londe, however, seems to have been put on the sidelines despite his international reputation, especially in the International Society of Rock Mechanics. The Malpasset dam failure could be recognised as the starting point of the discipline of rock mechanics in Civil Engineering (Londe, 1973). Londe analysed the dam failure in three dimensions using simple mechanics on a three-dimensional wedge that was believed to exist beneath the left abutment (note: convention in dams is to always look downstream in describing which half of the dam one is dealing with). The method is not dissimilar to the ‘Block Theory’ developed by Richard Goodman and Gen-Hua Shi (1985). Despite referring to Londe and Malpasset in their books (see also Goodman (1989)) they make no attempt at explaining the work done by Londe, Vigier and Vormeringer. Goodman and Shi offer one tantalizing clue, though, that their block theory should be used as the initial step in the process of analysing stability before continuing with the method of Londe, Vigier and Vormeringer: ‘It is beyond our present purpose to describe the solution of these parameters (shown in the plot) that determine the degree of safety of the wedge. We wish to point out, however, that such an analysis can only be run after a particular tetrahedral block has been singled out. Block theory is not a substitute for the limiting equilibrium analysis but, rather, a necessary prerequisite since it will allow you to determine which block to analyze’. Hence the purpose of this paper is to describe the solution.

STARTING, THEN, WITH BLOCK THEORY
Block theory requires plotting stereographic equal angle projection circles of the wedge planes beneath the dam and the plane of the slope (excavation pyramid). Note: many terms exist for ‘wedge’ such as ‘tetrahedral block’ as mentioned above or, used

ANALYSING THE ANALYSIS OF THE MALPASSET ARCH DAM FAILURE OF 1959
MICHIEL MAURENBRECHER DELFT UNIVERSITY OF TECHNOLOGY, SECTION GEO-ENGINEERING

At dusk on December 2 1959, engineers decided after heavy rains and an impending overflow of the reservoir to open the bottom valve outlet of the Malpasset concrete arch dam. Sixty metres of water pressure were then unleashed to try and lower the water levels in the reservoir. Three hours later the dam failed when the foundation of gneissic-schistose rock beneath the left abutment slid along a wedge. The reservoir water rushed towards the sea as almost sixty metres of flood water gushed through the completely obliterated left abutment portion of the dam. For eleven kilometres the water tore up farms, river banks and infrastructure as it headed towards the estuary of the Reyran River at Fréjus into the Mediterranean. 423 people died as a result of this disaster as well as flooding and destroying property and infrastructure along the path of the unleashed torrent.

FIGURE 1
CONTOUR MAP PRIOR TO FAILURE SHOWING THE PLAN OF THE DAM AND THE UNDERLYING WEDGE CAUSING FAILURE.
subsequently in Goodman & Shi (1985) to explain their block theory: ‘joint pyramid’. With three planes there are a possible eight wedges and in combination with an open slope face a ‘removable’ wedge can be defined. Removability means the wedge can move from the rock mass by sliding on one plane, two planes or no planes, the latter situation either by ‘popping’ or falling out. The wedge, though, will only move if the resultant force on the wedge acts outside a boundary or envelope defined by the geometry of the wedge and slope (region known as a ‘space pyramid’) and the angle of shear resistance of the wedge discontinuities. Londe did not carry out this first step as the field evidence after the failure defined the wedge surfaces and the wedge configuration from which the wedge slid or lifted. First though the geometry of the wedge has to be obtained. Surprisingly, one has to resort to a number of techniques to discover what parameters were used to analyse the stability of the dam! The following parameters were determined (after substantial ‘forensic research’ of the suite of ‘Londe papers’):

**Plane 1:** Upstream face of wedge ‘P1’, dip 45°, dip direction 270°.

**Plane 2:** Downstream face of wedge ‘P2’, dip 40°, dip direction 013°.

**Plane 3:** Toe face of wedge ‘P3’, dip 0°, dip direction horizontal.

The wedge in relation to the dam is shown on contour maps before and after failure in FIGURE 1 and 2. P1 and P2 are faults. P3 was an induced ‘crack’ (Londe, 1973).

For the Block Theory to establish removability, a slope (EP) is introduced based on the contour map in FIGURE 1, this is dip 33° with orientation 280°.

In FIGURE 3 the great circles are plotted of planes 1, 2 and 3 and the slope. These are plotted on an equal angle projection, in this instance using both the ‘upper focal point’ and the ‘lower focal point’. In the upper focal point projection the space within the reference circle (which coincides with the P3 great circle) is the ‘lower hemisphere’ and the ‘outer hemisphere’ is the space outside the reference circle. The three planes result in eight spherical triangles representing eight possible wedges or ‘Joint Pyramids’. Using Shi’s theorem (Goodman, 1989) the only wedge that does not intercept the slope great circle is JP 000. This is the wedge which is potentially removable and represents the wedge which was used to solve the stability of the dam foundations. There are two further wedges which are potentially removable should the orientation of the slope change and these are shown in FIGURE 3: JP 001 if the slope dips to 295° and JP 010 for a slope facing towards 265°. JP 000 means that the wedge planes are ‘upper half spaces’ of the discontinuities forming the planes. JP 001 means planes 1 and 2 are still ‘upper half spaces’ and plane 3 is a ‘lower half space’. In this instance plane 3 would be at higher elevation leaving an overhanging roof, the ‘upper half space’ in the rock mass. In the case of JP 010 planes 1 and 3 form ‘upper half spaces’ and 2 a ‘lower half space’. All three cases should be analysed though JP 010 is unlikely as the forces exerted on this wedge would probably ensure it remains in place.

The wedge would be stable if the resultant force on the wedge occurs within a zone defined by a spherical triangle with its intersection points at the poles of P1, P2 and P3. These are plotted in FIGURE 4 for the upper focal point projection (continuing from FIGURE 3).

The plot is further extended by connecting pole 1 with the intersection points of planes 1 & 2 and with planes 1 & 3. This is done for pole 2 with 1 & 2 and 2 & 3 and pole 3 with 2 & 3 and 1 & 2. The plots were all formed by generating the arcs using spherical trigonometry equations on a spreadsheet. Normally these are plotted by hand using an appropriate stereo net. The techniques used on a spreadsheet require a separate publication making use of spherical trigonometry and, in this instance, projection equations for an equal angle net (the x-scale and y-scale provide the projection equations to indicate the spacing in...
terms of $\alpha$). The Londe plots were all done by hand, probably using tracing paper placed over an extended equal angle stereonet of which an example is given in Goodman (1989). The advantage of a spreadsheet is that different values can be inserted to test the sensitivity of slope angle and direction, as well as difference in dip and dip direction for the discontinuity planes.

**FORCES ON A WEDGE: INFLUENCE OF FRICTION**

Stereographic projections are three-dimensional representations of angles. Forces have a direction but also a magnitude. One simplification making allowances for ‘magnitude’ in stereographic methods is to represent magnitude by use of the internal friction angle from the Mohr-Coulomb equation. Simply, this states that shear resistance is the product of the normal force acting on a surface times the tangent of the friction angle. Hence, if the direction of a force acting on a plane is known there could be a component parallel to the plane and a component normal to the plane ‘mobilising’ the frictional shear strength. Consider a force $F$ acting at an angle $\delta$ from the perpendicular to a plane. By resolving this force into a component along the plane, $F \sin \delta$, and a component normal to the plane $F \cos \delta$, and substituting this into the Mohr-Coulomb equation the mobilised shear resistance is $F \cos \delta \tan \phi$. If $F \cos \delta \tan \phi$ is greater than $F \sin \delta$ no sliding will occur. This relationship simplifies to $\tan \phi > \tan \delta$, and even further to $\phi > \delta$, leaving only angles to deal with.

The safe zone defined by the spherical triangle $p_1, p_2$ and $p_3$ is expanded by plotting the ‘friction cones’ at $p_1, p_2$ and $p_3$. If the force direction falls within the cone ($\delta < \phi$) sufficient shear resistance is mobilised to prevent movement. In FIGURE 5 the friction cones have been added and connected to each other at the end between the poles as the force changes direction it acts less on one plane and may start to act on two. Londe et al. (1969) and Londe (1973) describe the various types of sliding on one plane, on two planes along the line of intersection and possible dislodge-ment without sliding (or if in the opposite direction, where the wedge would be pushed into the rock mass, compression only). This results in seven modes of movement. Much of these two papers are devoted to this aspect. Goodman (1989) projection lines show where these zones can be found as well, but he has not defined them as such. The essential is if the resultant force falls in a zone that can cause movement or remain stable: a ‘safe zone’ and an ‘unsafe zone’. The Goodman approach and that of Londe are as good as identical. Goodman does have a preference for using the ‘lower focal point’ whereas Londe has used the ‘upper focal point’, hence in FIGURE 3 both projections are given. In FIGURE 4 the ‘upper focal point’ is used so that direct comparison can be made with the original Londe publications.

Further friction angle isolines have been added as was done by Londe showing that the stable zone increases substantially with increase in friction up to the great circle of planes 1, 2 and 3 for $\phi=90^\circ$.

**PLOTTING FORCES**

Two types of forces are dealt with: the forces exerted by the dam on the foundation (including the dead weight of the wedge) and water pressure (seepage) forces acting perpendicular to the wedge planes. The forces from the dam are the weight of the dam and the hydraulic forces of the reservoir acting on the dam. The magnitude and direction of these forces are provided in the 1970 paper but do not specifically refer to the Malpasset Dam.

The total weight, including the weight of the portion of the dam resting on the rock volume, is $W=111 000$ ton. The thrust of the dam is horizontal: $Q=84 000$ ton. The water forces corresponding to full hydrostatic head are:

1. $U_{1T}=85 000$ ton; 2. $U_{2T}=62 000$ tons; and (3)
U3T = 25 500 ton.
The above paragraph is taken verbatim from the paper. Q is shown in a direction 150° using the wedge diagram (reploted in FIGURE 1). The ‘dip’ of the resultant W with Q would be: tan⁻¹(111 000/84 000)=53°.

When this is plotted in FIGURE 5 (150°/53°), the resultant is in the ‘safe zone’. If the friction is reduced to zero, sliding of the wedge would occur on plane 3 only (zone Z3). As with the orientation of the planes, the Londe papers do not provide these values; the vector does correspond well with the projections (1970 and 1973 papers).

The final step is to examine the influence of U1T, U2T and U3T on the direction of F vector (point f). At this stage the final scenario of the analysis has been reached and to give Londe and his co-authors credit, the approach then and even today 30 years later has remained unique. A paper by Karaca & Goodman (1993) does indicate a similar approach showing the rotation of f under influence of the build-up of water pressures. Though they do not refer to any of the Londe papers, they could possibly have unwittingly presented the cause of the Malpasset Dam failure in their paper! A hint to this is given in the opening sentence of this paper.

**PLOTTING WATER FORCES**

To understand the Londe approach, the first paper (1969) presents a figure (reproduced here in FIGURE 6 for the water pressure force vectors on planes P1, P2 and P3). The amount of rotation of the force F when combined with forces U1, U2 and U3 individually is calculated for different percentages of U1T, U2T and U3T. The resulting zone constructed on the stereograph will show that not the full percentage of the water force is necessary to cause the resultant to occur in the ‘unsafe’ zone. These values are plotted in

---

**FIGURE 6**


---

**FIGURE 7**

**PLOTTING INFLUENCE OF WATER PRESSURES. FIRST STAGE: F TO U1T, F TO U2T AND F TO U3T TOWARDS POLES P1U, P2U AND P3U (UPPER HEMISPHERE, THE DIRECTIONS OF THE WATER FORCES ON P1, P2 AND P3 RESPECTIVELY). GREAT CIRCLES ARE ShOWN. THE ‘DISTANCES’ F TO U1T, F TO U2T AND F TO U3T ARE EQUAL TO THE ANGLES DETERMINED IN FIGURE 6. INTERMEDIATE VALUES OF 20%, 40%, 60% AND 80% ARE ShOWN. SECOND STAGE IS PLOTTED FOR F & U1T WITH U3T (POINT U13T) AND F & U2T WITH U3T (POINT U23T) SHOWING GRADUATIONS AGAIN FOR INTERMEDIATE PRESSURES 20 THROUGH 80%. COMPLETION OF SECOND STAGE AND FINAL THIRD STAGE TO POINT U123T IS ShOWN IN FIGURE 8, THIS TIME SOLELY BY GRAPHICAL TECHNIQUES OF INTERSECTING GREAT CIRCLES.**
ANALYSING THE ANALYSIS OF THE MALPASSET ARCH DAM FAILURE OF 1959

FIGURE 7 and plotted along the great circles common to f and the poles $p_1$, $p_2$ and $p_3$. These poles are located on the upper hemisphere of the stereograph (the outer portion) as they represent the directions of the water forces $u_1$, $u_2$, and $u_3$ (perpendicular to the planes P1, P2 and P3 in an upward direction as if to lift the underside of the wedge or, strictly, acting on the upper half spaces of the wedge discontinuities).

The water pressures do not, of course, build up individually on the wedge planes but build up gradually on one plane and possibly more rapidly on another depending on their permeability and drainage paths. In FIGURE 8, graphical methods were used to complete the unsafe zone for water pressures. The total vector positions (100% build-up) are combined by rotating $f_{-u_1T}$ in a direction $u_2$ to produce $f_{-u_2T}$ and then rotating $f_{-u_2T}$ in a direction $u_3$ to produce the point position $f_{-u_3T}$. In this way a zone is created, appropriately shaded indicating the range of positions where a resultant $F$ with $U(R)$ would cause the wedge to be unsafe and show which mode of movement would occur.

LAST WORD ON THE CAUSE OF THE MALPASSET DAM FAILURE?

This article hopefully explains the analysis and model used to examine the cause of failure of the Malpasset Dam. It not only condenses the original suite of papers by Londe but compiles pertinent aspects of the Londe suite essential to the analysis. The analysis is, however, not ‘the last word’ with regard to exploring the cause of failure. Londe et al. (1970) also looked at moment stability of the wedge as the forces developed on the wedge do not pass through a common point. This analysis showed that the wedge was not safe. Further analysis was carried out by Wittke & Leonards (1987) using finite elements resulting in another explanation as to the cause of failure though broadly it is still the discontinuities and water pressure build up that caused failure. What none of these analyses considered was what could have been the influence of the water spout under 60 m of water head when the bottom valve outlet was opened that fateful evening 48 years ago. Dubbed for the time being as the ‘Karaca-Goodman effect’ is the rise in discontinuity water pressures as a result of an impinging water jet. It is not the momentum of the water splashing against rock blocks which causes them to dislodge but the rise in discontinuity water pressures the water splashing induces. Could this be the trigger that initiated instability of the left abutment wedge at 9 pm that night of December 2, 1959?

REFERENCES


REPRINTED FROM The Ingeokring Newsletter, double issue 2007 / 2008, used with permission.