In 1960, Kalman published a paper in which he describes a filter to estimate the state of a linear system each time data becomes available. Later on, approximate filters for non-linear systems have been developed and used in the field of geomechanics for non-linear state space estimation. In a conceptual case study, based on the construction of a road embankment, the performances of two filters, the Ensemble Kalman filter and the Unscented Kalman filter are described. The measurements of the vertical displacement at several observation points were used to improve the uncertainty in the model state and the Young's modulus *E*.

A COMPARISON OF THE ENSEMBLE KALMAN FILTER WITH THE UNSCENTED KALMAN FILTER: APPLICATION TO THE CONSTRUCTION OF A ROAD EMBANKMENT

ANNEKE HOMMELS[1] – AKIRA MURAKAMI[2] – SHIN-ICHI NISHIMURA[2]

INTRODUCTION

In 1960, Kalman published a paper in which he describes a recursive solution to the discrete data linear filtering problem (Kalman, 1960). In a filter, the state of the system is analyzed each time data becomes available.
Although the linear Kalman filter was designed for linear systems, it is possible to derive approximate filters using a linearization techniques for non-linear systems. The Extended Kalman Filter is such a filter and has been used in the field of geomechanics for nonlinear state space estimation (Murakami, 1991). Over the last few years, a couple of alternative approaches have emerged, namely the Ensemble Kalman filter (EnKF) and the Unscented Kalman filter (UKF).

Evensen introduced the EnKF in 1994 and the theoretical formulations as well as an overview of several applications are described in Evensen (2003). The EnKF was designed to resolve two major problems related to the use of EKF. The first problem relates to the use of an approximate closure scheme in the EKF (first order Taylor expansion). The second problem relates to the huge computational requirements associated with the storage and forward integration of the error covariance matrix P. For further details the reader is referred to the references. In the Ensemble Kalman filter, an ensemble of possible state vectors, which are randomly generated using a Monte Carlo approach, represents the statistical properties of the state vector. The algorithm does not require a tangent linear model, which is required for the EKF, and is very easy to implement.

The UKF was first proposed by Julier and Uhlman (1997) and further developed by Wan and Van der Merwe (2001). Instead of linearizing the functions as is done in the EKF, the unscented transformation uses a set of points, and propagates these points through the actual nonlinear function. The points are chosen such that their mean, covariance and possibly also higher order moments match the Gaussian random variable. The mean and the covariance can be recalculated from the propagated points, yielding more accurate results compared to the ordinary function linearization.

Both the EnKF and the UKF have their advantages and disadvantages. The good performance of the EnKF has been shown in Hommels et al. (2005). The UKF has not been introduced in the field of geomechanics.
In the following sections, the EKF, EnKF and the UKF are shortly introduced, a more extensive introduction is found in the original article. In a conceptual nonlinear casestudy, based on the construction of a road embankment, the performances of the EnKF and the UKF are compared to each other. The measurements of the vertical displacement at several observation points were used to improve the uncertainty in the model state and theYoung's modulus E.

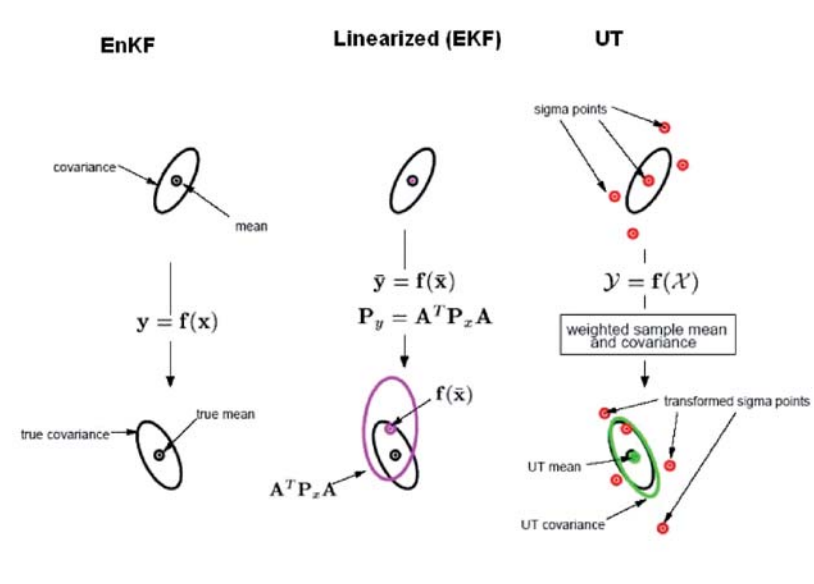


FIGURE 1
COMPARISON OF THE ENKF (MONTE CARLO), THE EKF AND THE UNSCENTED TRANSFORMATION, WHICH SERVES AS A BASIS FOR THE UKF (VAN DER MERWE ET AL., 2000)

KALMAN FILTERING

EXTENDED KALMAN FILTER

The linear Kalman filter is designed for linear systems. It is, however, possible to derive approximate filters using linearization techniques for non-linear systems.
Unscented Kalman filter

In 1997, Julier and Uhlmann introduced the Unscented Kalman filter (UKF) using the unscented transformation. Later, it was analyzed more in depth by Wan and Van der Merwe (2001). The UKF is based on the principle that is easier to approximate a Gaussian distribution than it is to approximate arbitrary nonlinear functions. They have shown that the UKF leads to more accurate results than the EKF and that the UKF generates much better estimates of the covariance of the states (the EKF seems to underestimate this quantity).

ENSEMBLE KALMAN FILTER

Evensen introduced the Ensemble Kalman filter in 1994 and the theoretical formulations as well as an overview of several applications are described in Evensen (1994 and 2003). The EnKF was designed to resolve two major problems related to the use of the extended Kalman filter, which are mentioned earlier.

In the Ensemble Kalman filter, an ensemble of possible state vectors, which are randomly generated using a Monte Carlo approach, represents the statistical properties of the state vector. The algorithm does not require a tangent linear model, which is required for the EKF, and is very easy to implement.

OVERVIEW

In *figure 1* a schematic diagram of the three different methods is shown. On the left, the EnKF has an ensemble of state vectors propagated through the non linear function f(x), and the true posterior mean and variance are calculated. In the middle, the EKF, the random variables are calculated by a linearization approach. On the right, the UKF is shown, where 5 sigma points are propagated through the non linear function f(x).

CONSTRUCTION OF A ROAD EMBANKMENT

Consider the construction of a four meter high embankment on soft clay (left of figure 2). A dedicated model for the determination of the settlement has been written using Smith and Griffiths (2004). In the left of *figure 2* the three-layered foundation below the embankment is shown. The nodes at which the observations of the vertical displacement were performed, are indicated with a triangle. The properties of the foundation, including the uncertain Young's modulii E are given in table 1. For the material model the purely elasto-plastic Mohr-Coulomb model is used. The groundwater table coincides with the original ground surface.

The embankment is built in four lifts and at the end of each lift a consolidation period is considered. The graph in the right of figure 2 shows at the left axis the different stages during the construction versus the total time, while at the right axis the measured displacement versus the total time at the end of each lift at a certain observation point, is shown. The measurements were generated based on an assumed true state.

TABLE 1 PROPERTIES OF THE FOUNDATION BELOW THE EMBANKMENT

Soil layer	μ_E (kPa)	σ_E	υ	c	φ	ψ	γ(kN/m³)	k_h(m/day)	k_v(m/day)
1	3400	340	0.33	16	12.5	0	16.5	2.41E-05	2.94E-05
2	1400	140	0.33	22	13.5	0	15.6	8.34E-05	8.99E-05
3	4600	460	0.30	8	6.5	0	17.3	3.56E-04	1.03E-04

RESULTS AND CONCLUSIONS

Figures 3, 4 and 5 show the results of the update

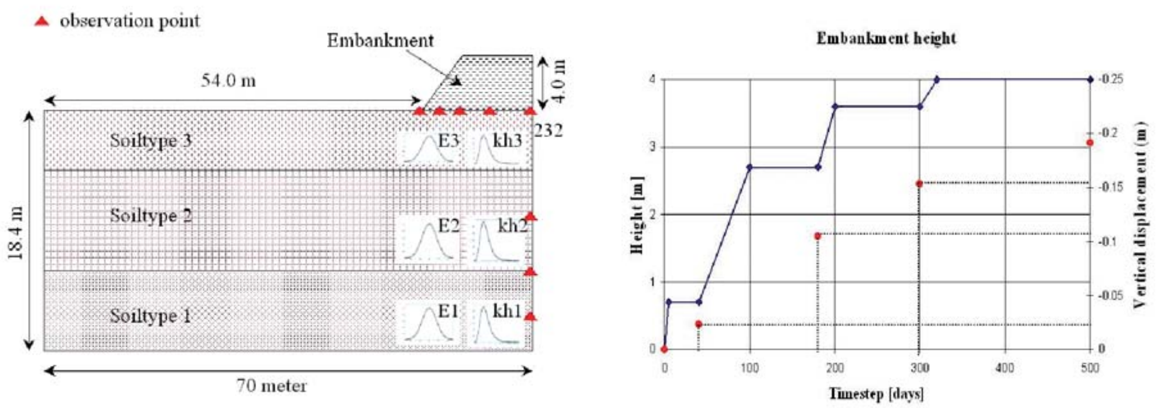


FIGURE 2
(LEFT) THREE-LAYERED FOUNDATION; THE OBSERVATION POINTS ARE INDICATED WITH A RED TRIANGLE; (RIGHT) EMBANKMENT HEIGHT (LEFT AXIS) AND VERTICAL DISPLACEMENT (RIGHT) DURING THE CONSTRUCTION PHASES

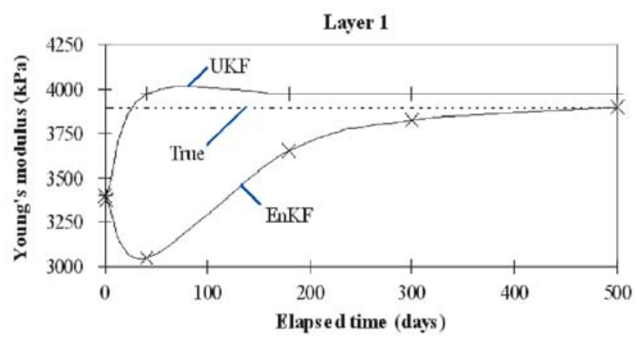


FIGURE 3
UPDATE OF YOUNG'S MODULUS E OF LAYER 1

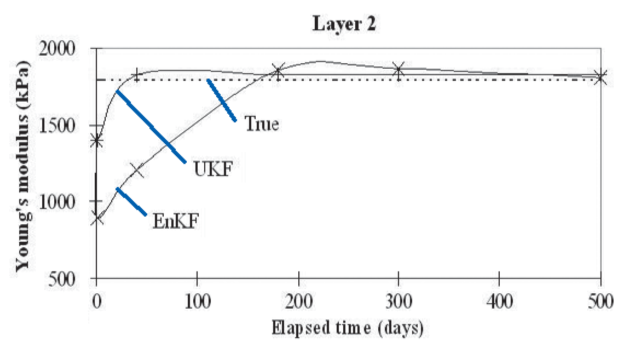
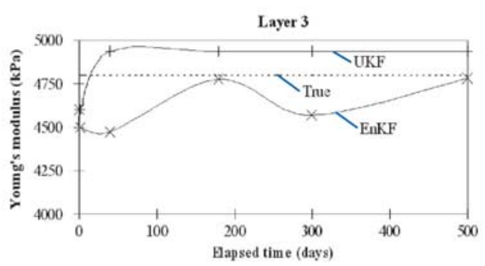


FIGURE 4
UPDATE OF YOUNG'S MODULUS E OF LAYER 2

process of the Young's modulii for soil layers 1,2 and 3 respectively using the EnKF and the UKF. The dotted horizontal line is the true value of the Young's modulus E in each layer. The results of the updated process using the EnKF are indicated with a '+'; the results of the updated process using the UKF are indicated with a 'x'.

From *figures 3 and 5* it is clear that the Ensemble Kalman filter performs much better than the Unscented Kalman filter. The UKF overestimates the Young's modulus in layers 1 and 3 and doesn't change much after one update, which probably implies that the filter is quite sure about its estimation. This can be seen in *figure 6*, which shows the standard deviation for each soil layer for the UKF. In figure 7, the standard deviation of the update process of the Young's moduli E of the three soil layers using the EnKF are shown. From *figure 7* it is clear that at the start of the update process the filter is not quite sure about his estimation, but at the end of the consolidation period, when the true state is reached, as can be seen in figures 3 and 5, the standard deviation reaches zero. Figure 4 shows more or less the same performance using the EnKF and the UKF; at the final update step the EnKF has a better value than the UKF, which

FIGURE 5
UPDATE OF YOUNG'S MODULUS E OF LAYER 3.



again is sure about his estimation (figure 6). This behaviour can be explained by the presence of only one observation point in layer 2, as can be seen in the left of figure 2.
From the above results it can be concluded that the EnKF performs much better than the UKF. For soil layers 1 and 3 this is more evident than for soil layer 2.

ACKNOWLEDGEMENTS

This research has been performed on the basis of the invitatio program for short period, provided by the Japanese Society for Promotion of Science (JSPS).
The authors are very grateful to the efforts of Satoshi Kasamutsu, who performed his MSc-thesis on this subject.

REFERENCES

– Evensen, G., *Sequential data assimilation with non-linear quasi-geostrophic model using Monte Carlo methods to forecast error statistics*, Journal of Geophysical Research, 99,143-162, (1994).
– Evensen, G., *The Ensemble Kalman filter: theoretical formulation and practical implementation*. Ocean dynamics, vol. 53, 4, p. 343-367 (2003).
– Julier, S.J. and J.K. Uhlman, *A new extension of the Kalman filter to nonlinear systems*. Proc. Of Aerosense, 11th Symp. On Aerospace/ Defense Sensing, Simulation and Control, (1997).
– Hommels, A., F. Molenkamp, A.W. Heemink and B. Nguyen, *Inverse analysis of an embankment on soft clay using the Ensemble Kalman Filter*. Proceedings of the Tenth Int. Conf. on Civil, Structural and Env. Eng. Computing, B.H.V. Topping (Editor), Civil-Comp Press, Stirling, United Kingdom, paper 252 (2005).
– Kalman, R.E., *A new approach to linear filter and prediction theory*. Journal of BasicEngineering, 82D, 35-45 (1960).
– Merwe, R. van der, A. Doucet, N. de Freitas and Eric Wan, *The Unscented Particle Filter*, Technical Report, TR380, Cambridge University, Engineering department (2000).
– Murakami, A., *Studies on the application of Kalman filtering to some geotechnical problems related to safety assessment*, dissertation, Kyoto University (1991).
– Smith, I.M. and Griffiths, D.V., *Programming the finite element method*, 4th Edition Chichester, Wiley and Sons, (2004).
– E.A. Wan and R. Van der Merwe, *The Unscented Kalman filter*, chapter 7 in Kalman Filtering and Neural Networks, Wiley, (2001). ■

[1] Anneke Hommels works at the Faculty of Civil Engineering and Geosciences, Delft University of Technology, The Netherlands.
[2] Akira Murakami and Shin-Ichi Nishimura are students at the Graduate School of Environmental Science, Okayama University, Japan.

Dit artikel is een verkorte versie van de inbreng van Anneke Hommels op de Europese conferentie voor Young Geotechnical Engineers (YGEC) in Gyor Hongarije. Zij is geselecteerd door en financieel ondersteund vanuit Kivi-Niria Geotechniek. Meer informatie over deze conferentie en het volledige artikel is te vinden op www.ygec2008.sze.hu.
De YGEC-Conferenties zijn bedoeld om beginnend geotechnici (onder 35 jaar) kennis te laten maken met internationale congressen, het vakgebied en netwerk.
De volgende YGEC wordt van 2-4 oktober gehouden direct voorafgaand aan het internationale congres van de ISSMGE in Alexandrië, Egypte. Kandidaten voor deelname kunnen worden aangemeld bij het bestuur van Kivi-niria Geotechniek.

FIGURE 6
STANDARD DEVIATION OF THE YOUNG'S MODULI DURING TIME OF THE THREE SOIL LAYERS USING THE UKF.

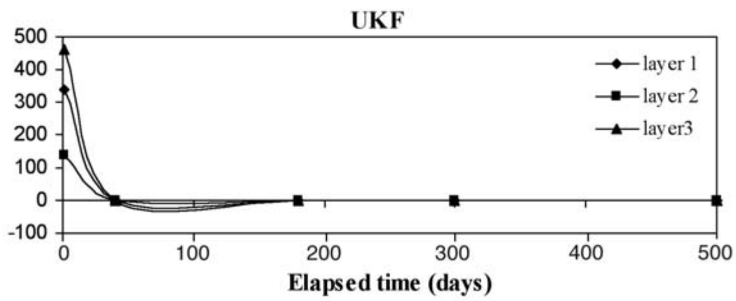


FIGURE 7
STANDARD DEVIATION OF THE YOUNG'S MODULI DURING TIME OF THE THREE SOIL LAYERS USING THE ENKF.